

STELLAR ORBITS AND THE INTERSTELLAR GAS TEMPERATURE IN ELLIPTICAL GALAXIES

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ABSTRACT

We draw attention to the close relationship between the anisotropy parameter $\beta(r)$ for stellar orbits in elliptical galaxies and the temperature $T(r)$ of the hot interstellar gas. For nearly spherical galaxies, the gas density ρ can be accurately determined from X-ray observations and the stellar luminosity density ℓ_* can be accurately found from the optical surface brightness. The Jeans equation and hydrostatic equilibrium establish a connection between $\beta(r)$ and $T(r)$ that must be consistent with the observed stellar velocity dispersion. Optical observations of the bright elliptical galaxy NGC 4472 indicate $\beta(r) \lesssim 0.35$ within the effective radius. However, the X-ray gas temperature profile $T(r)$ for NGC 4472 requires significantly larger anisotropy, $\beta \approx 0.6 - 0.7$, about twice as large as the optical value. This strong preference for radial stellar orbits must be understood in terms of the formation history of massive elliptical galaxies. Conversely, if the smaller, optically determined $\beta(r)$ is indeed correct, we are led to the important conclusion that the temperature profile $T(r)$ of the hot interstellar gas in NGC 4472 must differ from that indicated by X-ray observations, or that the hot gas is not in hydrostatic equilibrium.

Subject headings: galaxies: elliptical and lenticular, CD – galaxies: active – cooling flows – X-rays:
galaxies – galaxies: clusters: general – X-rays: galaxies: clusters

1. INTRODUCTION

The anisotropy of stellar orbits in elliptical galaxies can be estimated from the temperature of the hot interstellar gas through which they move. These two quite different galactic attributes are intimately related by the Jeans equation for the stars and the condition for hydrostatic equilibrium in the gas. This simple relationship is valuable since both the anisotropy and gas temperature are difficult to extract from optical and X-ray observations, respectively.

Many massive elliptical galaxies are nearly spherical (Merritt & Tremblay 1996) and slowly rotating, but their stellar velocity ellipsoids are not in general isotropic. The non-spherical nature of the stellar velocity dispersion is represented by the parameter

$$\beta = 1 - \sigma_t^2 / \sigma_r^2$$

where σ_r is the radial stellar velocity dispersion and σ_t is the dispersion in a transverse direction, i.e. $\sigma_t^2 = \sigma_\theta^2 = \sigma_\phi^2$. If the orbits are predominantly radial, $0 < \beta < 1$, the line of sight velocity profile becomes more strongly peaked than a Gaussian profile with increasing projected radius R ; if the orbits are mostly tangential, $-\infty \leq \beta < 0$, the profile is more flat-topped and becomes broader with increasing radius. Both $\beta(r)$ and the galactic potential $\Phi(r)$ can be determined from optical observations of the line of sight velocity dispersion $\sigma(R)$, the optical surface brightness distribution and the deviation of the stellar line profiles from a Gaussian as expressed by the line-symmetric coefficient $h_4(R)$ in a Gauss-Hermite expansion (e.g. van der Marel & Franx 1993).

Stellar line profiles observed in most massive E galaxies

indicate a preference for radial orbits with $\beta \sim 0.3$ (Bender, Saglia & Gerhard 1994; Gerhard et al. 1998; Saglia et al. 2000). Accurate $\beta(r)$ profiles from optical data require high quality data and considerable care in reduction and analysis. The radially anisotropic nature of stellar orbits in luminous elliptical galaxies provides important and otherwise unavailable information about the merger history of these galaxies (Naab, Burkert & Hernquist 1999), so an improved or independent determination of $\beta(r)$ would be desirable.

Similarly, the radial variation of the hot gas temperature $T(r) \sim T_{vir} \sim 10^7$ K in elliptical galaxies depends on the spatial and spectral resolutions of X-ray detectors and the accuracy of the three-dimensional decomposition that converts T as a function of projected radius R to physical radius r . In addition, it is often unclear whether the hot gas at any radius has a single temperature or a multitude of temperatures as might be expected if the gas were cooling. It is unclear if the gas cools at all. For example, high resolution X-ray spectra with XMM-Newton of the large E galaxy NGC 4636 fail to show emission lines expected from gas at intermediate temperatures, such as the 0.574 keV OVII line (Xu et al. 2002), suggesting that the gas is not cooling below $\sim T_{vir}/3$ as in classical cooling flows. However, Bregman, Miller & Irwin (2001) detected the OVI 1032,1038Å doublet in NGC 4636 emitted from gas at $T \sim 3 \times 10^5$ K, implying that cooling near the expected rate may occur after all. As an additional source of confusion, the observed X-ray spectra can often be significantly improved by assuming two quite different discrete temperatures at each galactic radius (e.g. Buote 2002 for NGC 1399; Buote et al. 2002a and Tamura et al. 2003 for NGC

5044). While the origin and physical nature of gas with only two (or a limited continuum of) temperature phases may be conceptually problematical, it may be related to certain heating mechanisms. Nevertheless, there are good reasons for suspecting the accuracy or interpretation of gas temperature profiles $T(r)$ in elliptical galaxies.

In the following discussion we illustrate the close relationship between the radial variations of $\beta(r)$ and $T(r)$ using data for the well-observed giant elliptical NGC 4472. While currently available data for this galaxy are adequate to illustrate this relationship, more accurate optical and X-ray data will be necessary to verify $\beta(r)$ or $T(r)$ with confidence.

We recognize that NGC 4472 is not perfectly spherical; its optical image has been variously classified as E1 or E2. However, optical determinations of $\beta(r)$ for NGC 4472 and other nearly-spherical elliptical galaxies have been made under the assumption of spherical symmetry (e.g. Kronawitter et al. 2000). Indeed the very concept of the β anisotropy parameter is only valid in spherical geometry. Similarly, the gas temperature $T(r)$ determined from X-ray observations of NGC 4472 and other nearly spherical elliptical galaxies are based on the assumption of spherical symmetry. One justification for this is that the stellar gravitational potential that confines the stars and hot gas within the half-light radius is always more spherical than the underlying stellar luminosity distribution (e.g. Brighenti & Mathews 1996).

Our discussion of the close relationship between stellar anisotropy and gas temperature is a thinly-veiled request for observers to concentrate on those luminous E galaxies for which the most accurate X-ray and optical data can be acquired.

2. RELATING $\beta(r)$ AND $t(r)$

The Jeans equation for the radial stellar velocity dispersion σ_r is

$$\frac{1}{\ell_*} \frac{d(\ell_* \sigma_r^2)}{dr} + \frac{2\beta \sigma_r^2}{r} = -g \quad (1)$$

where ℓ_* is the stellar luminosity density corresponding to the stellar velocity dispersions σ_r and σ_t . The equation for hydrostatic equilibrium in the hot interstellar gas is

$$\frac{1}{\rho} \frac{d(\rho c^2)}{dr} = -g. \quad (2)$$

Here $c^2 = kT/\mu m_p$ is the isothermal sound speed, m_p is the proton mass and $\mu = 0.61$ is the molecular weight. The uniformity of the stellar mass to light ratio within the effective radius of NGC 4472 is excellent evidence that the hot gas is in hydrostatic equilibrium (Brighenti & Mathews 1997). Both equations contain the same gravitational term $g = GM(r)/r^2$. Eliminating this term results in an equation explicitly independent of the confining mass:

$$\frac{d\sigma_r^2}{dr} + \frac{\sigma_r^2}{r} \left[\frac{d \log \ell_*}{d \log r} + 2\beta \right] = \frac{dc^2}{dr} + \frac{c^2}{r} \left[\frac{d \log \rho}{d \log r} \right]. \quad (3)$$

This equation has been discussed previously, particularly in the context of the anisotropic orbits of galaxies in rich clusters that contain hot gas (e.g. Fabricant et al. 1989). In addition to hydrostatic equilibrium, the validity of this equation depends on the assumption that the gas pressure exceeds that of any other interstellar pressure such as turbulence, cosmic rays, etc.

If $\beta(r)$ and $T(r)$ (as well as $\rho(r)$ and $\ell_*(r)$) are known from observations, then Equation (3) can be solved for $\sigma_r(r)$ and the line of sight velocity dispersion is then found from

$$\sigma^2(R) = \frac{\int_R^{r_t} \sigma_r^2(r) [1 - (R/r)^2 \beta] \ell_*(r) (r^2 - R^2)^{-1/2} r dr}{\int_R^{r_t} \ell_*(r) (r^2 - R^2)^{-1/2} r dr} \quad (4)$$

(e.g. Binney & Mamon 1982). In this equation R is the projected radius and r_t is some radius significantly beyond the limit of the available data (we assume $r_t = 3R_e$). If $\beta(r)$ is known, Equation (3) can be solved for $\sigma_r(r)$ for various parameterized temperature profiles $T(r)$ until $\sigma(R)$ from Equation (4) agrees with observed velocity dispersions. By this means it is possible to derive the gas temperature profile $T(r)$ from a knowledge of $\beta(r)$. Conversely, if the gas temperature $T(r)$ is securely known from X-ray observations, then $\beta(r)$ can be determined by requiring that solutions $\sigma^2(R)$ of Equations (3) and (4) pass through the observations. This close relationship between $\beta(r)$ and $T(r)$ is possible and valuable because $\ell_*(r)$, $\rho(r)$ and $\sigma^2(R)$ are much easier to determine from observations than either $\beta(r)$ or $T(r)$.

Except for the very central region, the stellar brightness in NGC 4472 is well-fit with a de Vaucouleurs profile. The stellar B-band luminosity density $\ell_*(r) \equiv \ell_{*,deV}(r)$ is determined by the total luminosity $L_B = 7.89 \times 10^{10} L_{B,\odot}$ and the effective radius: $R_e = 104'' = 1.733' = 8.57$ kpc, assuming a distance $d = 17$ Mpc. However, within a small “break” radius, $r_b = 2.41'' = 200$ pc, the stellar luminosity flattens to $\ell_{*,core}(r) = \ell_{*,deV}(r_b)(r/r_b)^{-0.90}$ (Gebhardt et al. 1996; Faber et al. 1997). In Figure 1 we compare the (un-normalized) stellar luminosity density $\ell_*(r)$ in NGC 4472 with the hot gas density profile $\rho(r)$ determined from X-ray observations. The remarkable proportionally $\ell_*(r) \propto n_e^2(r)$ was first discovered by Trinchieri, Fabbiano & Canizares (1986).

X-ray observations of the hot gas temperature in the central region of NGC 4472 are shown in Figure 2. Only three observations of the gas temperature are available in the region within 10 kpc where β has been determined from optical observations. The filled circles show ROSAT deprojected gas temperature observations from Buote (2000). The linear temperature profile defined by these two temperature observations

$$T(r) = 0.7480 \times 10^7 + 7.251 \times 10^5 r_{kpc} \quad (5)$$

is shown in Figure 2 with a solid line. From Chandra data Soldatenkov, Vikhlinin & Pavlinsky (2003) have recently provided an additional temperature measurement of $T/10^7$ K = 0.766 ± 0.023 at radius $1'' = 0.0824$ kpc that is shown with a filled square in Figure 2. The linear fit to the Buote data passes within the error bars of the Soldatenkov et al. observation, so all three observations are consistent with a linear temperature profile within the effective radius $R_e = 8.57$ kpc. Although the X-ray data is sparse in the region of interest, it is in perfect qualitative agreement with linear gas temperature profiles observed in other bright elliptical galaxies in $r \lesssim R_e$ (Fig. 1 of Brighenti & Mathews 1997).

2.1. Determining $\beta(r)$ from $T(r)$

The open circles in Figure 3 show observations of the line of sight stellar velocity dispersion in NGC 4472 (from Bender et al. 1994) that were used by Kronawitter et al. (2000) to determine $\beta(r)$ from optical observations alone. The filled circles in Figure 3 are the data of Fried & Illingworth (1994). Both sets of observations lie entirely within the effective (half-light) radius, $R_e = 104'' = 8.57$ kpc. For some reason the normalization is systematically different in the two sets of optical observations. The Fried-Illingworth data have smaller error bars, less overall scatter and resemble in overall form the velocity dispersion profiles $\sigma(R)$ of other well-observed ellipticals [e.g. Gerhard et al. (1998) for NGC 6703]. The observational uncertainties apparent in the data sets shown in Figure 3 make it difficult to determine with confidence any radial dependence of the stellar anisotropy. Consequently, we assume that the orbital anisotropy β in Equation (3) is constant over the range $r > r_o = 0.34$ kpc.

In Figure 3 we compare solutions of Equations (3) and (4) with the observed line of sight stellar velocity dispersion in NGC 4472. Each solution curve $\sigma(R)$ is based on the linear gas temperature profile in Figure 2 and a particular value of β . More information about the solutions shown in Figure 3 can be found in Table 1. Of the three solid lines designed to fit the Fried-Illingworth data, solution 2 with initial $\sigma_r(r_o) = 430$ km s⁻¹ and $\beta = 0.7125$ fits the data best. Solutions 1 and 3 show additional marginally acceptable fits although they fall respectively below and above the Fried-Illingworth data at smaller projected radii R . Evidently $\beta = 0.71 \pm 0.15$ is the most likely anisotropy based on the gas temperature observations. In some of our solutions of Equation (3) $\sigma_r^2(r)$ becomes unphysically negative at a radius r_z less than the outer limit of our integration in Equation (4), $r_t = 3R_e$; for these solutions we replace r_t with r_z . In solutions 4 - 6, shown with dashed lines in Figure 3, we find that the anisotropy parameter must have values $\beta = 0.63 \pm 0.15$ to be in approximate agreement with the data of Bender et al. (1994). Once $\sigma_r(r_o)$ is selected for each of these dashed line solutions in Figure 3, the vertical position of the solution is extremely sensitive to β , i.e., β must be specified to one part in 1000 to pass through the center of the Bender et al. data. One may doubt if Nature can be so discriminating. Nevertheless, for either set of optical data, the corresponding values of β are about twice as large as the $\beta(r) \lesssim 0.34$ found by Kronawitter et al. (2000) from the Bender et al. data.

2.2. Determining $T(r)$ from $\beta(r)$

The preferred orbital anisotropy in NGC 4472 determined by Kronawitter et al. (2000) from the Bender et al. observations can be accurately represented with

$$\beta_K(r) = 0.449r_{kpc}^{0.7333} \exp(-r_{kpc}/2.472),$$

which peaks at $r \approx 1.65$ kpc where $\beta_K \approx 0.34$. In this section we assume that $\beta_K(r)$ is correct and solve Equation (3) for $c(r)^2$ and $T(r)$. As before we consider only linear temperature variations defined by two temperatures T_1 and T_2 evaluated respectively at $r_1 = 2.18$ kpc and $r_2 = 8.55$ kpc. We seek values of T_1 and T_2 for which solutions $\sigma(R)$ of Equation (4) pass through or near the Bender et al. optical data.

We begin with a set of solutions for which $\beta = \beta_K(r)$ and the temperature profile is constrained to agree with the observed linear $T(r)$ in Figure 2. When $\ell_*(r)$ and $\rho(r)$ are taken from Figure 1 as before, the only remaining undetermined parameter required to solve Equations (3) and (4) is $\sigma_r(r_o)$. The four short dashed lines in Figure 4 show the line of sight velocity dispersion profile $\sigma(R)$ for $269.10 \leq \sigma_r(r_o) \leq 269.25$ km s⁻¹, corresponding to solutions 7 - 10 in Table 1. Since we are using the Kronawitter et al. $\beta_K(r)$, it is appropriate to compare these solutions only with the data of Bender et al. from which $\beta_K(r)$ was derived. Of these, the best-fitting solution is 9, but the overall flat or positive slope [$d\sigma(R)/dR > 0$] at larger R does not resemble the $\sigma(R)$ profiles of most well-observed E galaxies. Since none of these solutions is a particularly good fit to the Bender et al. data, it appears that $\beta_K(r)$ found from optical data alone is somewhat inconsistent with the observed hot gas temperature profile.

To explore alternate X-ray temperature profiles that may agree better with the $\sigma(R)$ of Bender et al., we consider solution 11 in which the hot gas is isothermal at the mean value of the two Buote temperatures in Figure 2, $T_1 = T_2 = 1.137 \times 10^7$ K. This solution, shown with a dotted line in Figure 4, decreases slightly with R , in better agreement with data in $R \lesssim 2$ kpc. Solutions with negative dT/dr may fit even better, but all known X-ray observations for E galaxies have positive dT/dr for $r < R_e$. Finally, we consider solution 12 based on a temperature profile having the same slope as the observations in Figure 2, but with uniformly higher temperatures, $T_1 = 1.306 \times 10^7$ K and $T_2 = 1.768 \times 10^7$ K. In solution 12 (solid line in Figure 4) the radially decreasing line of sight dispersion $\sigma(R)$ follows the Bender et al. data better than any other curve in Figure 4, possibly suggesting that the gas temperature should be higher than the observations in Figure 2. Therefore, either the gas temperatures from X-ray observations in Figure 2 are too low or $\beta_K(r)$ and the $\sigma(R)$ data from which it was derived are inaccurate.

3. SUMMARY AND FINAL REMARKS

We have shown that Equations (3) and (4) establish a useful correspondence between the anisotropy of stellar orbits in elliptical galaxies $\beta(r)$ and the temperature of the hot interstellar gas $T(r)$. In practice, the $\beta - T$ relation is based on the assumption of hydrostatic equilibrium for the hot gas and requires reliable observations of the stellar and gas densities and the stellar line of sight velocity dispersions. This connection between β and T is possible only if the galaxies are nearly spherical, as emphasized by Maggorian & Ballantyne (2002).

If the gas temperature profile $T(r)$ in an E galaxy is securely known from X-ray observations, $\beta(r)$ can be determined from the observed stellar velocity dispersion using Equations (3) and (4). In principle, this method of determining $\beta(r)$ could be done at galactic radii much larger than R_e where stellar observations of $h_4(R)$ may be difficult but where observations of $\sigma(R)$ and X-ray gas temperatures are generally reliable. Conversely, if $\beta(r)$ can be observed with high precision from optical observations, the hot gas temperature $T(r)$ can be found from Equations (3) and (4) by using the gas density from X-ray observations. This means of determining $T(r)$ may be particularly

valuable in regions of the hot gas atmospheres where X-ray spectra indicate a two-temperature or limited multi-temperature thermal structure (e.g. Buote 2002b) that may complicate the determination of the average local gas temperature.

As a representative example, we have used the $\beta-T$ relation to determine $\beta(r)$ in the bright elliptical galaxy NGC 4472 from the hot gas temperature profile $T(r)$ found from X-ray observations. The resulting anisotropy, $\beta \sim 0.71 \pm 0.15$, indicates that the stellar orbits are considerably more radially biased than implied by the smaller $\beta_K(r) \lesssim 0.3$ determined from optical observations alone (Kronawitter et al. 2000). These large β for NGC 4472 are based on the Fried-Illingworth stellar velocity dispersions $\sigma(R)$. Somewhat lower anisotropies, $\beta = 0.63 \pm 0.15$ are consistent with the velocity dispersion observations of Bender et al., but these β still exceed the $\beta_K(r) \lesssim 0.3$ found optically by Kronawitter et al. (2000). Alternatively, if the Kronawitter et al. $\beta = \beta_K(r)$ and the Bender et al. (1994) data are correct, then the gas temperature in NGC 4472 may be higher than previously thought from X-ray observations and possibly have a different slope dT/dr . These conclusions apply only to the limited region $r < R_e$

for which $\sigma(R)$ data currently exist for NGC 4472.

It is clear, at least for NGC 4472, that currently available observations are not sufficiently accurate to establish either $\beta(r)$ or $T(r)$ with complete confidence. Accurate optical observations of $\sigma(R)$ and stellar line profiles can be observed with 8 - 10 meter telescopes to several R_e and $T(r)$ is also often known at these large radii. In addition, it may be possible to determine gas temperature profiles with more accurate or prolonged X-ray observations than those in Figure 2. The $\beta(r)$ determined from X-ray emission can serve to calibrate the accuracy of purely optical determinations or to provide a test for hydrostatic equilibrium. To accomplish this, it would be desirable to obtain high quality stellar dispersion and gas temperature observations of those galaxies – such as NGC 4472, NGC 4649, NGC 1399, M87, etc. – that are luminous at both optical and X-ray frequencies.

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FIG. 1.— The observed and azimuthally averaged electron density n (cm^{-3}) profile in NGC 4472 as a function of radius normalized to the effective radius $R_e = 8.57$ kpc (distance $d = 17$ Mpc). The observations are from *Einstein* (Trinchieri, Fabbiano, & Canizares 1986) (*filled circles*) and *ROSAT* (Irwin & Sarazin 1996) (*open circles*). For the inner region we have Abel-inverted Chandra surface brightness data from Loewenstein et al. (2001) (*open squares*) and normalized them to previous observations. The solid line is an analytic fit to the observations. The dashed line is the square root of the stellar luminosity density $\ell_*^{1/2}(r)$ with arbitrary vertical normalization.

FIG. 2.— X-ray observations of the gas temperature in NGC 4472 based on deprojected *ROSAT* observations of Buote (2000) (*filled circles*). Also shown with a filled square is the Chandra observation of the gas temperature at 0.082 kpc from Soldatenkov et al. (2003). Note that the central temperature is nonzero. The solid curve is a linear fit for $T(r)$ that passes through all three observed temperatures.

FIG. 3.— The line of sight stellar velocity dispersion as a function of projected radius. The observations are from Bender et al. (1994) (*open circles*) and Fried & Illingworth (1994) (*filled circles*). The lines show constant β solutions based on the gas temperature $T(r)$ variation shown in Figure 2. Each line type corresponds to several numbered solutions listed in Table 1 where values of $\sigma(r_o = 0.34$ kpc) or $\sigma(R_e = 8.57$ kpc) can be used to uniquely identify each line in the plot.

FIG. 4.— The line of sight stellar velocity dispersion as a function of projected radius. The data are the same as in Figure 3. The lines show solution curves based on the orbital anisotropy $\beta_K(r)$ from Kronawitter et al. (2000). The numbers associated with each curve refer to solutions listed in Table 1. Each short dashed line can be uniquely identified with a solution in Table 1 by comparing values at $\sigma(R_e) = 8.57$ kpc.

TABLE 1
SOLUTIONS OF EQUATIONS (3) AND (4)

Solution	β^a	T_1^b (10^7 K)	T_2^b (10^7 K)	$\sigma_r(r_o)$ (km s $^{-1}$)	$\sigma_r(R_e)$ (km s $^{-1}$)	r_z^c (kpc)	$\sigma(r_o)$ (km s $^{-1}$)	$\sigma(R_e)$ (km s $^{-1}$)
$T \Rightarrow \beta$								
1	0.6333	.906	1.368	390.	249.	25.6	282.	172.
2	0.7125	.906	1.368	430.	253.	23.7	293.	162.
3	0.7775	.906	1.368	470.	267.	$> 3R_e$	306.	171.
4	0.4766	.906	1.368	330.	276.	$> 3R_e$	306.	295.
5	0.63	.906	1.368	390.	290.	$> 3R_e$	318.	270.
6	0.773	.906	1.368	470.	326.	$> 3R_e$	334.	245.
$\beta \Rightarrow T$								
7	β_K	.906	1.368	269.10	205.	$> 3R_e$	247.	210.
8	β_K	.906	1.368	269.15	219.	$> 3R_e$	272.	281.
9	β_K	.906	1.368	269.20	232.	$> 3R_e$	295.	337.
10	β_K	.906	1.368	269.25	244.	$> 3R_e$	317.	385.
11	β_K	1.137	1.137	330.38	254.	$> 3R_e$	315.	303.
12	β_K	1.306	1.768	332.9	258.	$> 3R_e$	316.	295.

^a β_K refers to $\beta(r)$ from Kronawitter et al. (2000).

^bThe linear temperature variation is defined by values T_1 at $r_1 = 2.18$ and T_2 at $r_2 = 8.55$ kpc.

^cThe radial velocity dispersion $\sigma_r(r)$ goes to zero at r_z .







